
# Math

# Understand the contents

We a glad to present to you **Gimnasio virtual San Francisco Javier.** Present it presented through texts for elementary education program and the sequence mathematically oriented contents, enriched by videos and subtopics.

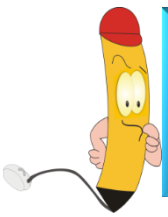
Handle this text thoughtfully to acquire new attitudes, skills, abilities and concepts which will allow you to expand your worldview.

Contents are grouped into four units containing four sessions of topics and subtopics consisting of several pages. Each topic begins with a title, a series of questions whose purpose is to inspire your interest in studying the topics. You can use these questions at the end to make a self check, how much do you know now.

You'll find images related to the concepts and themes, videos, charts, concept maps to ensure that you understand the logic behind concepts.

The funny helpers are there to support you in your journey through this adventure of knowledge.

## Let us search...

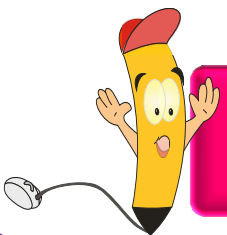


When you find this pencil, you will know that there were many questions without answers. You can use them at the end of a topic to find what you have learned.

When you find this helper, you have to carry out the activities for each topic or subtopic

Art is a part of your activities, giving a personal touch making it in color.
Now you are the artist!

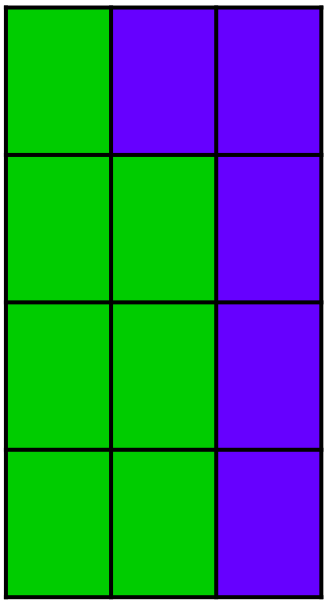
Teacher helper invites you to learn more about the topic, to study new things. It is becoming very interesting to know!



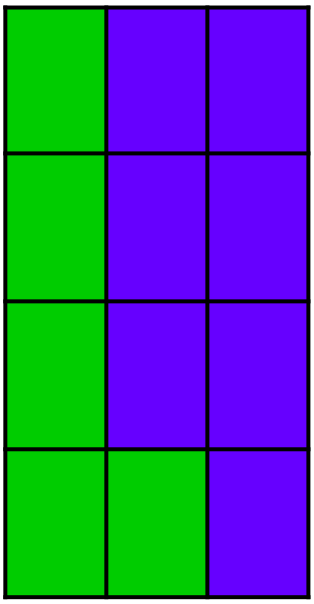
**Did you know that...?**
You'll find fun facts that invite you to learn more about other related topics.

# Comparing fractions

Which is bigger: $\frac{7}{12}$ or $\frac{5}{12}$ ? http://www.youtube.com/watch?v=lEQtJ_dTtRM



$\frac{7}{12}$



$\frac{5}{12}$

Fraction $\frac{7}{12}$ is greater than fraction $\frac{5}{12}$

Symbolize: $\frac{7}{12} > \frac{5}{12}$

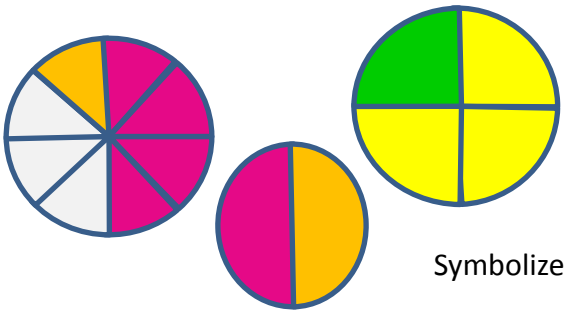
For fractions with the same denominator the greater (bigger) fraction has bigger numerator (7 > 5).

# Operations with Fractionss

http://www.youtube.com/watch?v=xOhbX-1tORo

## Multiplying fractions

1. What is the fourth part of a half?

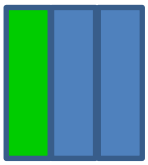


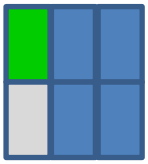
The fourth of one half is **eighth.**

Symbolize: $\frac{1}{4} \times \left(\frac{1}{2}\right) = \frac{1}{8}$

A quarter of a half.

2. What is half of a third?



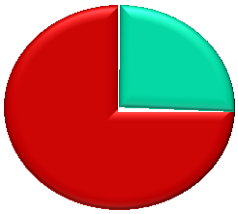


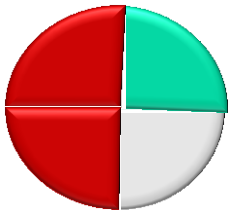
A half from a third

"**Half**" of third is a sixth.

Symbolize: $\frac{1}{2} \times \left(\frac{1}{3}\right) = \frac{1}{6}$

3. How much is two thirds of three quarters'?





The "two-thirds" of three fourths are two quarters.

Symbolize: $\frac{2}{3} \times \left(\frac{3}{4}\right) = \frac{6}{12}$

Apply the operator $\frac{2}{3}$ **x ( )** to three quarters, means applying

Initially the operator thirds $\frac{1}{3}$ **x ( )**, then the operator double: **2 x ( ).**

"The fourth part" of one half is "eighth" and "third" of eighth is "three eighths".

Multiplication of fractions means multiplying together numerator by numerator and denominator by denominator.

Remember. Dividing numerator and denominator by the same number does not alter the value of a fraction. This process is called simplification.

The result of $\frac{2}{5} \times \frac{3}{7} = \frac{6}{35}$

6 Is obtained: multiplying 2 × 3.
35 Is obtained: multiplying 5 × 7.

¿ $\frac{6}{35}$ can be simplified? NO.

The result of $\frac{4}{9} \times \frac{3}{5} = \frac{12}{45}$

4 × 3 = 12 and 9 × 5 = 45

$\frac{12}{45}$ Can be simplified by 3: $\frac{12 \div 3}{45 \div 3} = \frac{4}{15}$

So:

$$\frac{4}{9} \times \frac{3}{5} = \frac{12}{45} = \frac{4}{15}$$

Do not forget to simplify fractions whenever possible. Before, during and after multiplication.

# Addition and subtraction of fractional numbers

Addition or subtraction of fractions with the same denominator.

To add fractions with like denominators, add the numerators.

$$\frac{5}{14} + \frac{6}{14} = \frac{5+6}{14} = \frac{11}{14}$$

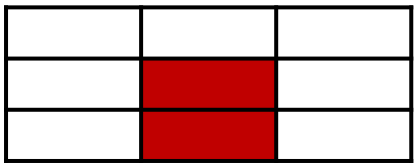
The 11/14 part of the wall is colored.

This wall was colored as follows: 1/7 of green, 2/7 of blue and 3/7 of yellow. Which part of the wall is painted?

$$\frac{1}{7} + \frac{2}{7} + \frac{3}{7} = \frac{1+2+3}{7} = \frac{6}{7}$$

The 6/7 part of the wall is painted.

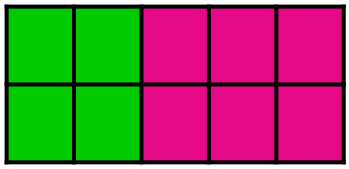
It is necessary to paint with red 7/9 of the table. Paul painted 2/9. Which part is not painted yet?



$$\frac{7}{9} - \frac{2}{9} = \frac{7-2}{9} = \frac{5}{9}$$

5/9 of the table is not painted yet

Its necessary to paint the board with green. Ana painted 3/10 and Hugo painted 1/10. Which part of the board is still unpainted?



Painted part:

$$\underbrace{\frac{3}{10}}_{\text{Ana}} + \underbrace{\frac{1}{10}}_{\text{Hugo}} = \frac{3+1}{10} = \frac{4}{10} = \frac{2}{5}$$

Unpainted part:

$$\frac{5}{5} - \frac{2}{5} = \frac{5-2}{5} = \frac{3}{5} = \frac{3}{5}$$

- $\frac{5}{5}$: Complete board
- $\frac{2}{5}$: Painted part
- $\frac{5-2}{5}$: Unpainted part
- $\frac{3}{5}$: Not painted yet

Remember to add or subtract fractions with the same denominator, you have to add or subtract the numerators. Then place the denominator. If possible, simplify the answer.

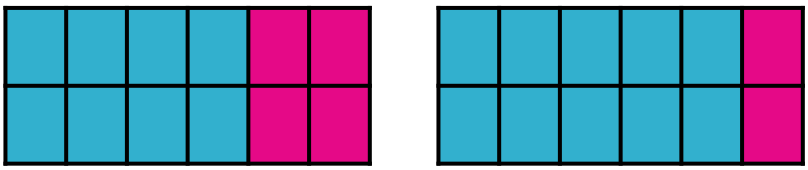
# Addition or subtraction of fractions with different denominators

To add fractions with different denominators they must be transformed into equivalent fractions with equal (common) denominator.

The common denominator is the LCM (Least Common Multiple) of denominators.

LCM is a correct mathematical term, but Least Common Denominator (LCD) also is used.

How much is $\frac{2}{3} + \frac{5}{6}$?



$\frac{2}{3}$ and $\frac{5}{6}$

They have different denominators!
To add them we must find a common denominator.
**Note: 6 is a multiple of 3.**

Then complete the fraction $\frac{2}{3}$ multiplying both, numerator and denominator by 2.

$$\frac{2}{3} \times \frac{2}{2} = \frac{4}{6} \text{ and } \frac{4}{6}, \frac{5}{6}$$

$$\frac{2}{3} + \frac{5}{6} = \frac{4}{6} + \frac{5}{6} = \frac{9}{6}$$

$$\frac{3}{3} + \frac{4}{9} + \frac{5}{6}$$

These fractions have different denominators!
Remember: the common denominator is the LCM of the denominators.

# Decimal numbers

Bobina comes with 50 meter of rope. 9.5 meters has been used to tie boxes and 6.25 meters in tying a parcel. We still have:
http://www.youtube.com/watch?v=cxxEVKGEICI

a. 34.25 m
b. 34 m
c. 24.25 m
d. 15.75 m

2. The decimal number equivalent to the fraction $\frac{17}{5}$ is:

a. 0.17
b. 1.7
c. 0.34
d. 3.4

3. Five hundred seven hundredths is written :

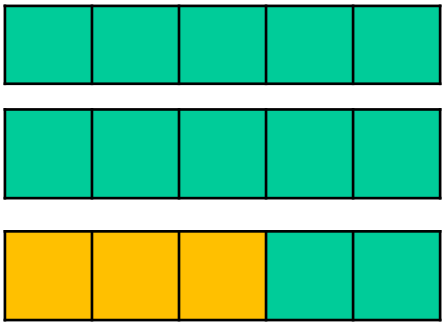
a. 0.507
b. 5.07
c. 0.0507

d. $\frac{507}{100}$

# Mixed numbers

## Returning to fractions

The fractional part of the unit is obtained by dividing the unit into equal parts.
http://www.youtube.com/watch?v=A5axf-0Uhgo



The representation corresponding to the fraction $\frac{12}{5}$

This fraction is greater than 1, since its numerator 12 is greater than the denominator 5.

We can write:

$$\frac{12}{5} = \frac{10+2}{5} = \frac{10}{5} + \frac{2}{5} = 2 + \frac{2}{5}$$

The expression $2 + \frac{2}{5}$ can be shortened: $2\frac{2}{5}$

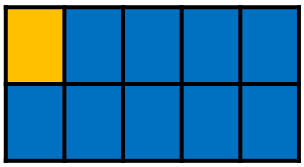
And it reads : "Two and two-fifths."

2 corresponds to the integer part and 2/5 to the fractional part.

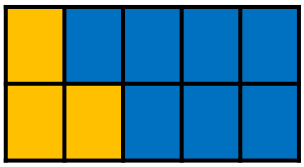
Fractions where both integer and fractional part are present are called **mixed numbers.**

# Decimal fractions

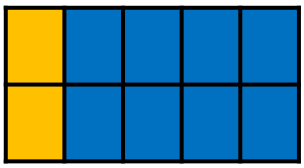
Fractions whose denominator is **a one followed by zeros** are called decimal fractions.



$\frac{1}{10}$ « One Tenth»

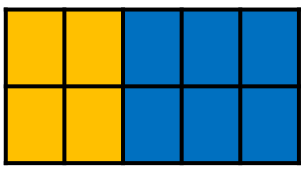


$\frac{3}{10}$ "Three tenths"

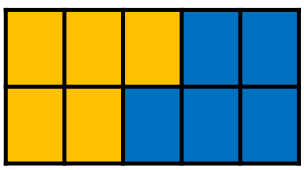


$\frac{2}{10}$ "Two tenths"

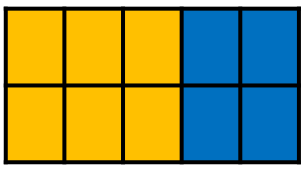
All these fractions have 10 as a denominator.



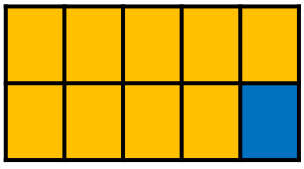
$\frac{4}{10}$ "Four tenths"



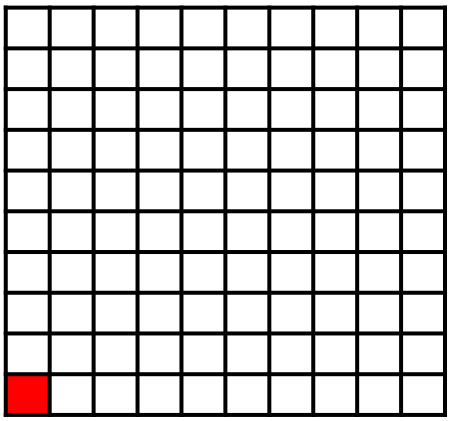
$\frac{5}{10}$ "Five tenths"



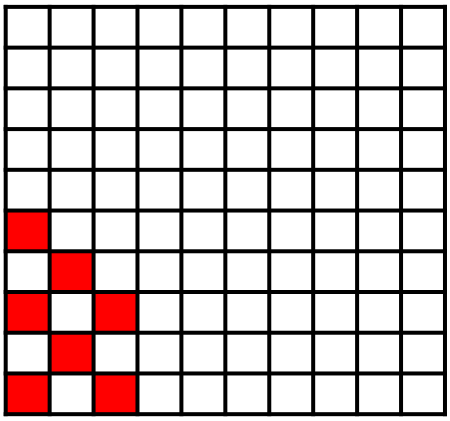
$\frac{6}{10}$ "Six-tenths"



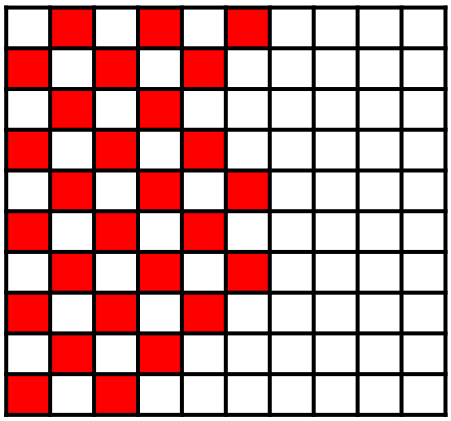
$\frac{9}{10}$ "Nine-tenths"



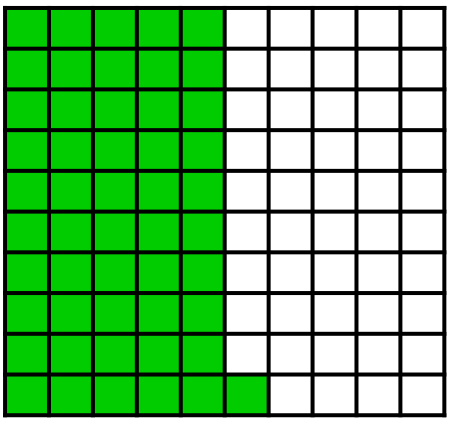
$\frac{1}{100}$ " One Hundredth"



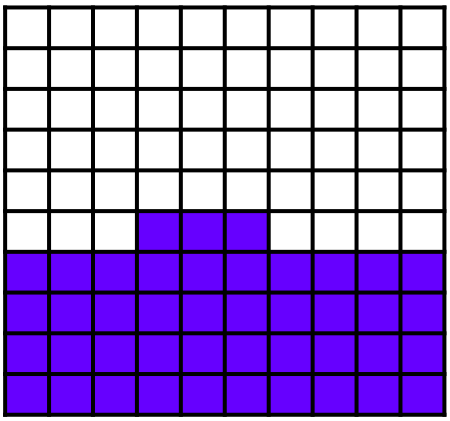
$\frac{7}{100}$ "Seven hundredths»



$\frac{27}{100}$ "Twenty-seven hundredths»



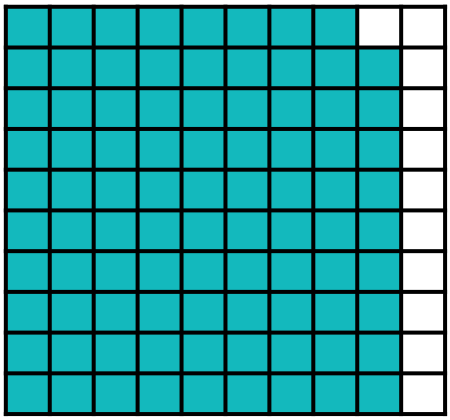
$\frac{51}{100}$ «fifty-one hundredths »

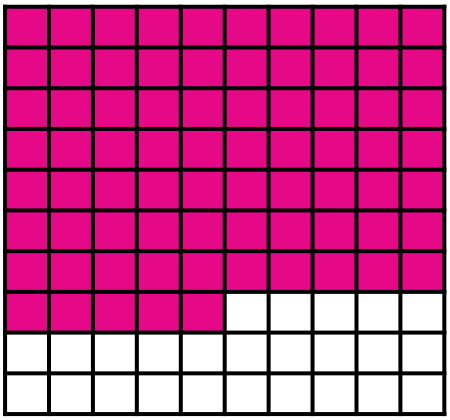


All these fractions have 100 as denominator.

$\frac{43}{100}$ "Forty-three hundredths»

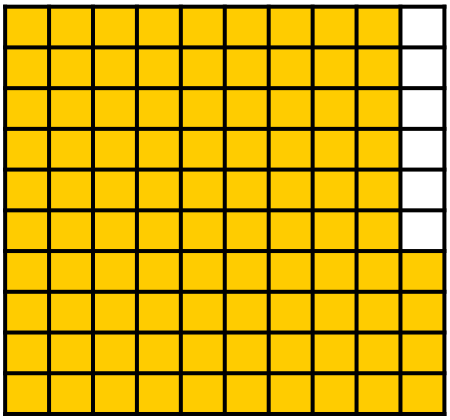
The fractions with denominators 10, 100, 1000, or en other words they are multiples of 10 are known as decimal fractions.





$\frac{89}{100}$ ____

$\frac{51}{100}$ ____



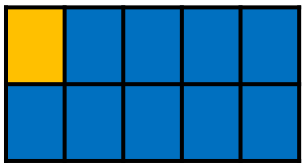
$\frac{94}{100}$ ____

Decimal fractions:

$\frac{102}{1000}$ $\frac{93}{1000}$ $\frac{72}{100}$ $\frac{13}{10}$

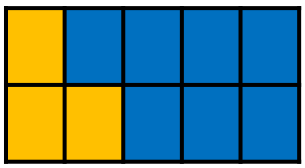
# Decimal numbers equivalent to decimal fractions

Decimal fractions can be written as **decimal numbers. A decimal number is another way of expressing a fractional number.**



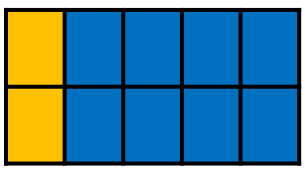
$\frac{1}{10}$

«Tenth» = 0.1 tenth



$\frac{3}{10}$

«Three tenths »= 0.3 three tenths



$\frac{2}{10}$

"Two tenths" = 0.2 two tenths

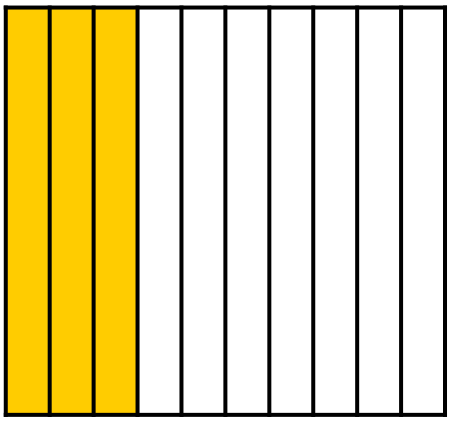
$\frac{7}{10} = 0\frac{7}{10} = 0.7$

An integer and fractional parts of decimal numbers are separated by point (.).

# Decimal equivalents

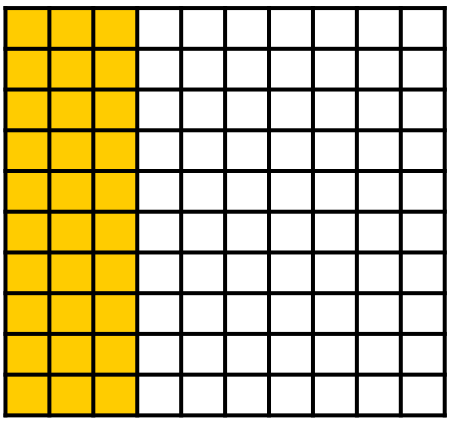
Ending zeros from a decimal number can be deleted or added without altering it.

Every natural number can be expressed as a decimal number with any number of zeros after decimal point.



$\frac{3}{10}$ → 0.3

Three tenths



$\frac{30}{100}$ → 0.30

"Thirty hundreds»

$\frac{3}{10}$ and $\frac{30}{100}$

are equivalent, because 3 x 100 = 30 x 10.
Then 0.3 and 0.30 are equal decimals.

# Adding decimals

Decimal numbers are placed so that the unit positions match. To avoid confusion, you can add missing zeros.

Then add them like natural numbers. Write the decimal point in the result after the units place.

**3.5 + 14.23 + 7 = 24.73**

| | Thousand units | Hundreds | Tens | Units | Tenths | Hundredths | Thousandths |
|---|---|---|---|---|---|---|---|
| | | | | 3. | 5 | **0** | |
| + | | | 1 | 4. | 2 | 3 | |
| | | | | 7. | **0** | **0** | |
| | | | 2 | 4. | 7 | 3 | |

1. Calculate **324.25 + 1385.75.**

Write the numbers aligning the decimal point. Proceed with summation and write the decimal point in the right place.
The result can be read «one thousand seven hundred ten».

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| | | **3** | **2** | **4.** | **2** | **5** | |
| + | 1 | 3 | 8 | 5. | 7 | 5 | |
| | **1** | **7** | **1** | **0.** | **0** | **0** | |

# Subtraction of decimal numbers

- Place properly the subtrahend under the minuend.
- Subtract as integer numbers.
- Place the decimal point in the result after units position.

**28.05 – 17.98 = 10.07**

| | Thousand units | Hundreds | Tens | Units | Tenths | Hundredths | Thousandths |
|---|---|---|---|---|---|---|---|
| | | | 2 | 8. | 0 | **5** | |
| - | | | 1 | 7. | 9 | 8 | |
| | | | **1** | **0.** | **0** | **7** | |

1. Calculate **321.3 – 196.87.**

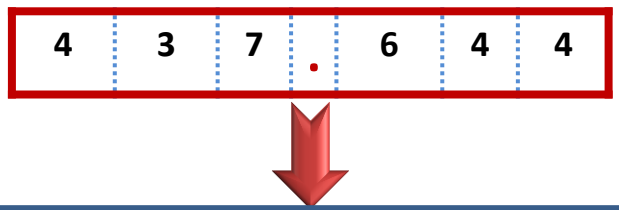
- Line up the decimal points of the numbers.
- You can write zeros in the minuend to align all digits.
- Subtract and place the decimal point in the difference.

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| | | **3** | **2** | **1.** | **3** | **0** | |
| - | | 1 | 9 | 6. | 8 | 7 | |
| | | **1** | **2** | **4.** | **4** | **3** | |

# Multiplication of decimal numbers

- Operation is performed like it is made with natural numbers.
- The product digits are separated by (.) counting from the right a number of decimal places equal to the sum of decimal places in both factors.

| | Thousand units | Hundreds | Tens | Units | Tenths | Hundredths | Thousandths |
|---|---|---|---|---|---|---|---|
| | | | 9 | 5. | 1 | 4 | |
| X | | | | 4. | 6 | | |
| + | | 5 | 7 | 0 | 8 | 4 | |
| | 3 | 8 | 0 | 5 | 6 | | |
| | 4 | 3 | 7 | 6 | 4 | 4 | |




You put the decimal point as the number of decimal places show you

# Multiplication and division of decimal numbers by 10, 100 or 1000.

To multiply a decimal by 10, 100 or 1000, move the decimal point to the right as many places as many zeros have the multiplier.

**5.823 x 100 = 582. 3**

**Since 100 has two zeros, move the decimal point two places to the right.**

## Dividing a decimal by a natural number

- The process of division a decimal by a natural number is similar to that of the natural numbers.
- When you get to the place of decimal point in the dividend, write one in the quotient.

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| 3 | 4 | 9 | . | 8 | 6 | | | |
| 3 | 0 | | | | 5 | 8 | . | 3 |
| | 4 | 9 | | | | | | |
| | 4 | 8 | | | | | | |
| | | 1 | | 8 | | | | |
| | | 1 | | 8 | | | | |
| | | | | 0 | | | | |

To divide a decimal by 10, 100 or 1,000 move the decimal point to the left as many places as many zeros have the divisor.

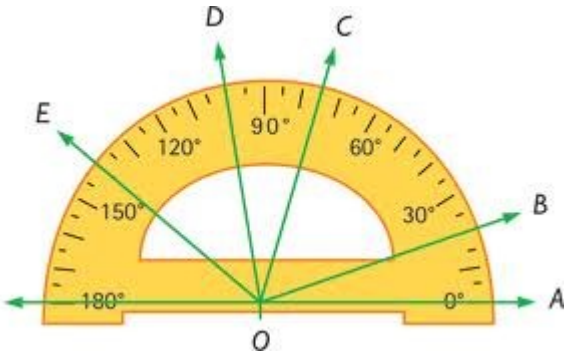
**689.71 ÷ 10 = 68.971**

**As 10 has one zero, you move the point once.**

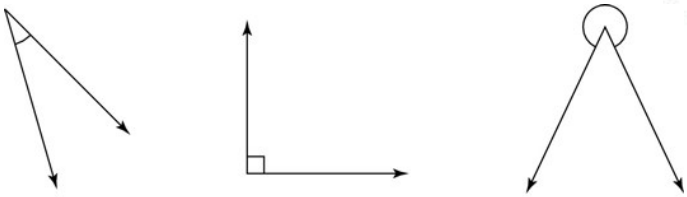
# Length and geometry

## Angles

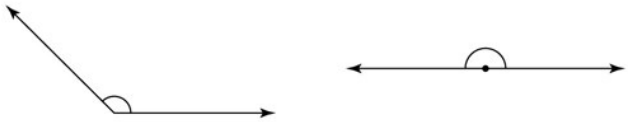
The angle elements are the initial and terminal sides and vertex.



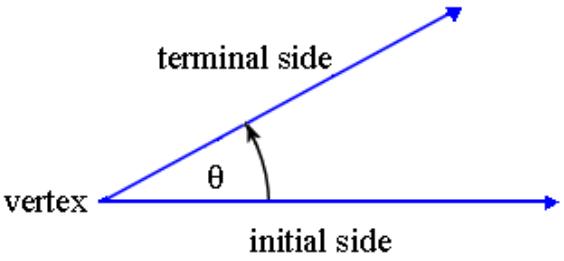

For measure is used the transporter.



Acute angle | Right angle | Reflex angle



Obtuse angle | Straight angle



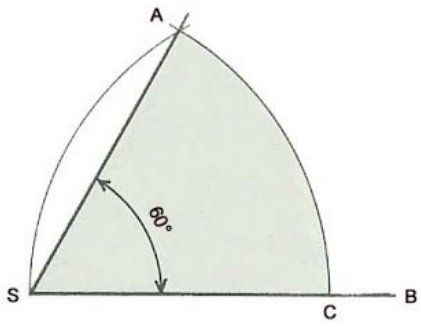

- Read, analyze and understand how to construct an angle with certain measures.

1. Draw a ray and mark the point of origin P.


2. Place the transporter so that:

•The point P coincides with the center of the transporter
•The ray passing through the down side.

3. Search 60° at transporter's scale degrees. Point it out with a small dot.
4. Take a ruler and connect initial point P with a dot mark.

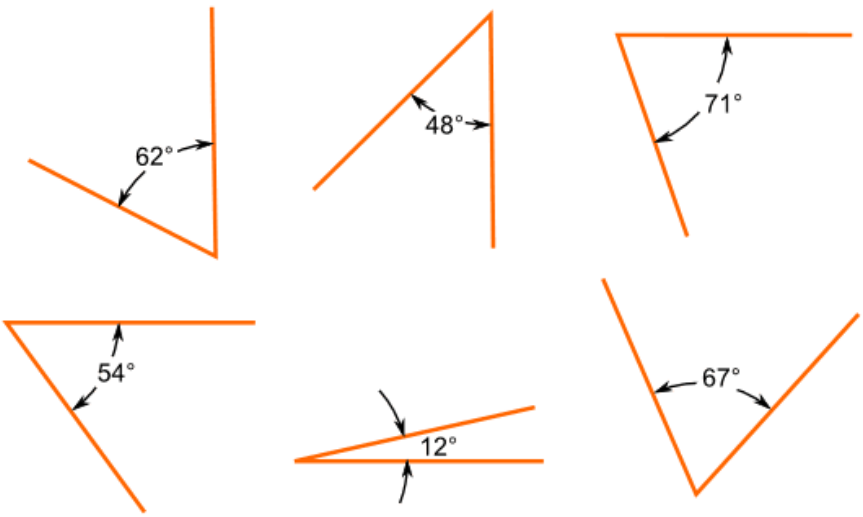



60° degree angle.

# Classification of the angles

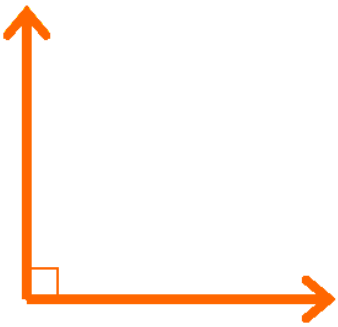
## Acute

•Measures less than 90° degrees



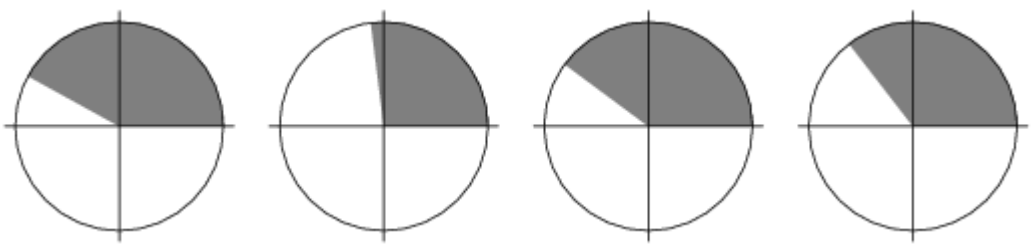

## Right

- It measures 90°.



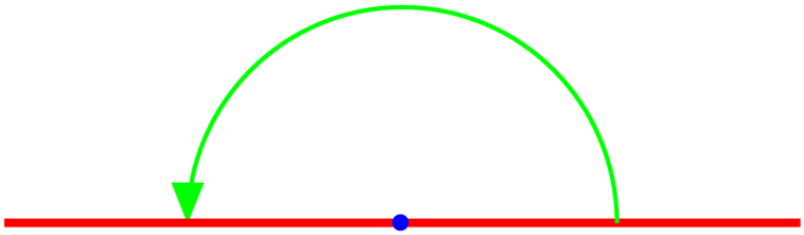
## Obtuse

It is more than 90 degrees and less than 180 degrees.



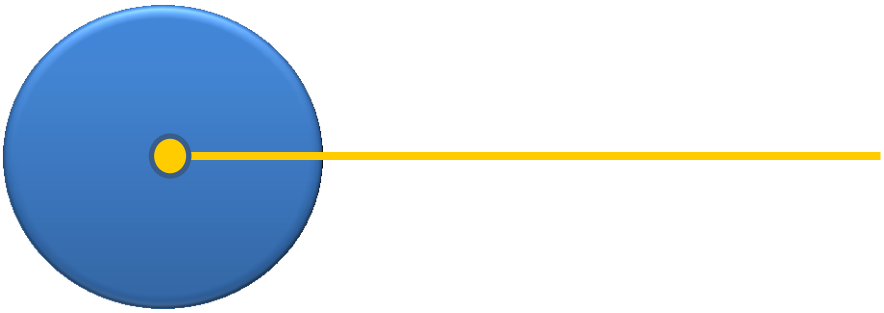
## Straight

It measures 180°.



## Complete

Its measurement is 360°. One full turn.



The angle value is written in the following form: first comes special symbol to denote angle, then goes the name of the angle (small letter of Greek alphabet are widely used) and equality sign followed by numeric value.

$$\angle \alpha =$$

The angle is a geometric figure formed on a surface by two lines starting from the same point.

We can also say that the opening angle is formed by two rays called sides, which have a common origin called the vertex.

# Classification of polygons

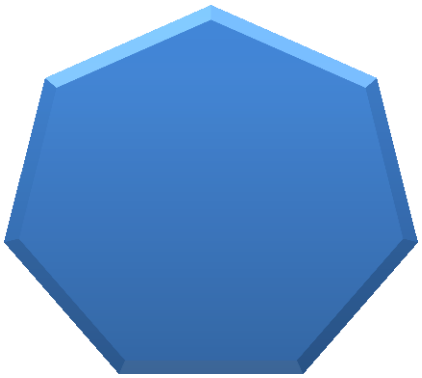
Polygons can be classified according to the number of sides and by their shape.

| Number of sides | Figure |
|---|---|
| 3 | Triangle |
| 4 | Quadrilateral |
| 5 | Pentagon |
| 6 | Hexagon |
| 7 | Heptagon |
| 8 | Octagon |

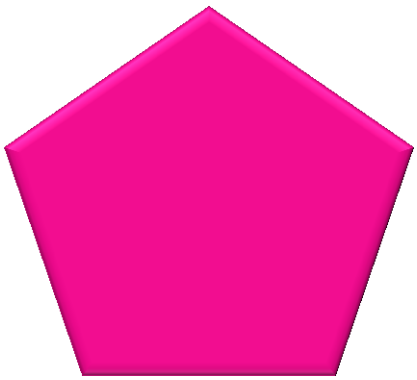
## The perimeter of polygons.

The perimeter of a polygon: The sum of the lengths of its sides. The perimeter can be calculated in regular and irregular polygons with measuring instruments.
When measuring the perimeter of a regular polygon, a square or an equilateral triangle, simply measure one side and multiply the result by the number of sides that have the figure. Remember that these polygons are characterized by equal sides.

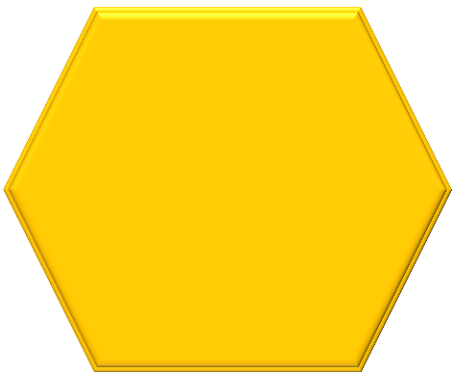
When the polygon is irregular, ie having different side lengths, we need to measure each side and then calculate their sum to obtain the perimeter of the figure.



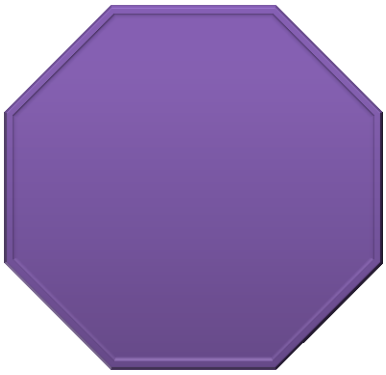
Heptagon



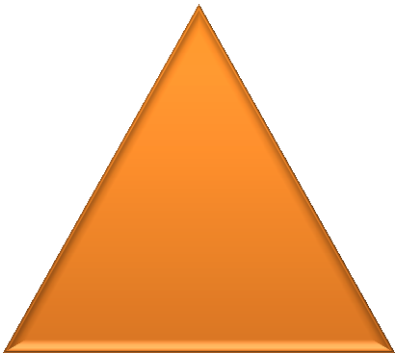
Pentagon



Hexagon



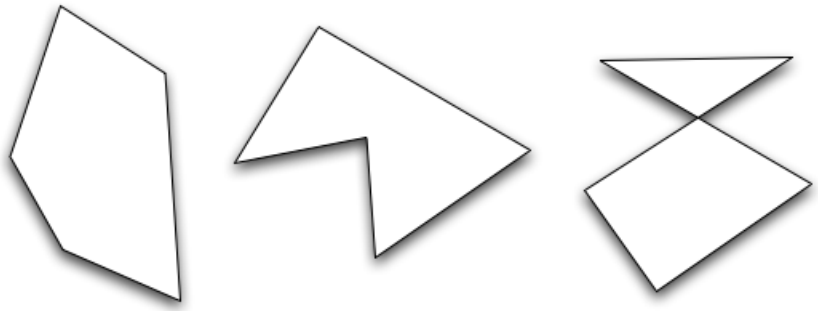
Octagon



Triangle

Quadrilateral

# Shape of polygons



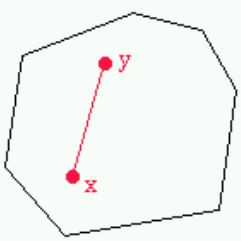
Convex polygon  Concave polygon  Self-intersecting polygon

A polygon is **convex** if:

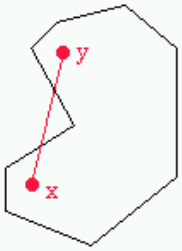
•All of its angles are less than 180°.
•All of its diagonals are internal .

A polygon is **concave or non-convex**

If:
• It has an angle greater than 180°.
• One of its diagonals lies outside.




A convex polygon




A non–convex polygon

# Parallelograms

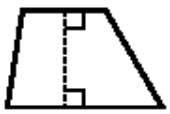
A parallelogram is a quadrilateral with two pairs of parallel sides.

## Properties

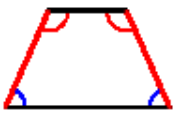
1. In all parallelograms opposite angles are equal and adjacent angles to the same side are supplementary.
2. In all parallelograms the opposite sides are equal.
3. In any parallelogram the diagonals intersect each other in equal ratio.
4. The diagonals of a rectangle are equal.
5. The diagonals of a rhombus are perpendicular bisectors with right angle.
6. The diagonals of a square are equal, perpendicular bisectors of its angles.

Not all quadrilaterals are parallelograms, some quadrilateral having a single pair of parallel opposite sides and called trapeze and others which have no pair of parallel opposite sides and called trapezoids.

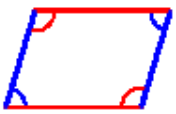
*Trapezium*
*(Amer. Eng.)*



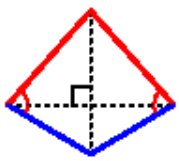
*Trapezoid (Amer. Eng.)*
*Trapezium (Brit. Eng.)*



*Isosceles trapezoid (Am.)*
*Isosceles trapezium (Br.)*

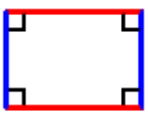


*Parallelogram*

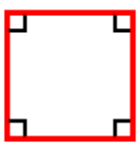


*Kite*

*Rhombus*



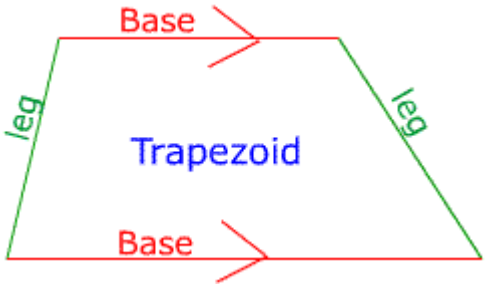
*Rectangle*



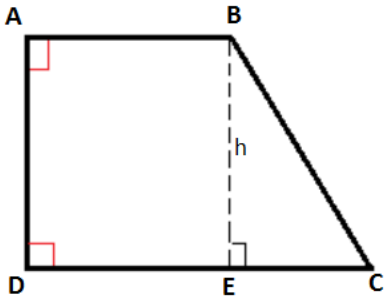
*Square*

# Trapezoids

A trapezoid is a quadrilateral with one pair of parallel sides




## Right trapezium

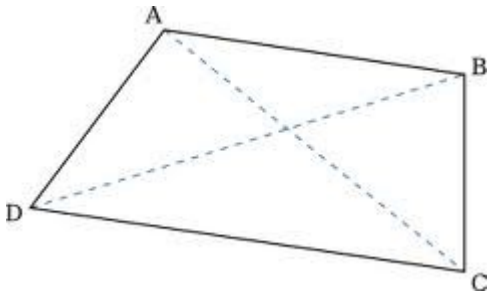



Right Trapezium

Trapezoid with one leg at right angle with base. Recall that bases are parallel. Thus right trapezium have two right angles.

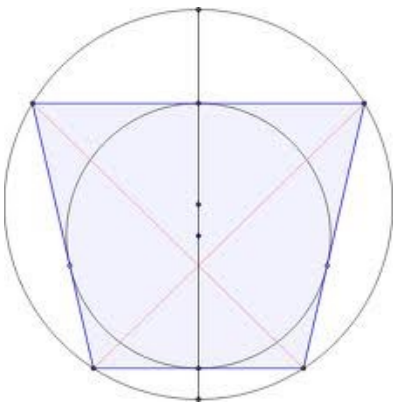
## Scalene trapezoid

Trapezoid does not have equal sides or angles.

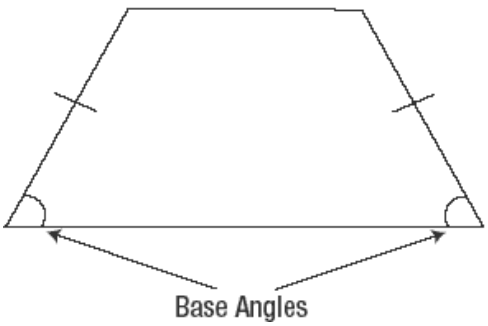



## Isosceles trapezoid

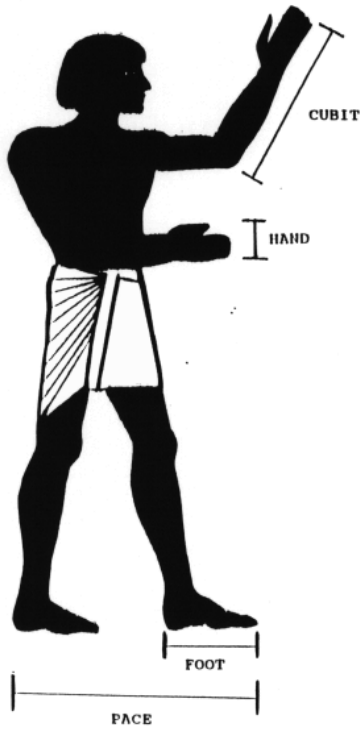
Trapezoid with a line of symmetry bisecting one pair of opposite sides.



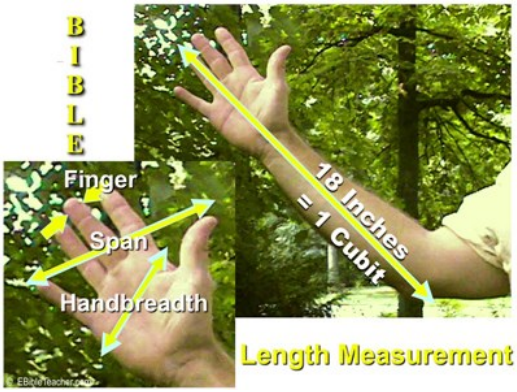
Two opposite sides (the bases) are parallel, and the two other sides (the legs) are of equal length. The diagonals are also of equal length. The base angles of an isosceles trapezoid are equal in measure.

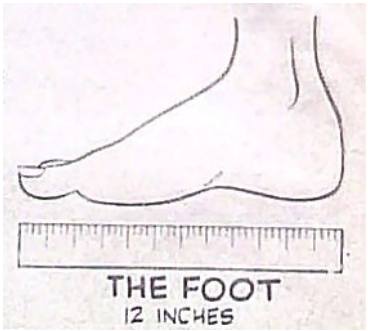


# Length



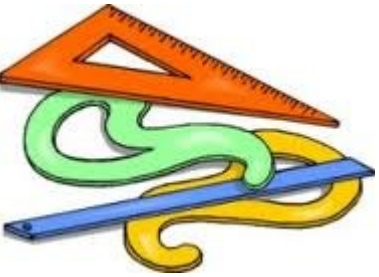

In the history of mankind people used many types of length measures, a lot of them were based on the size of the human body or its parts. They were not exact and were replaced by different measurement standards.

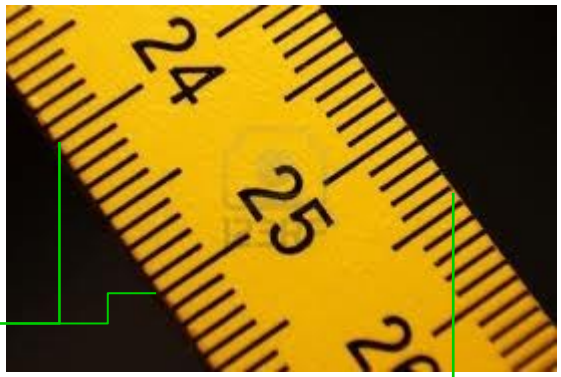



## Using the ruler

The ruler is a tool that helps us to draw straight lines and measure lengths in centimeters, millimeters or inches.

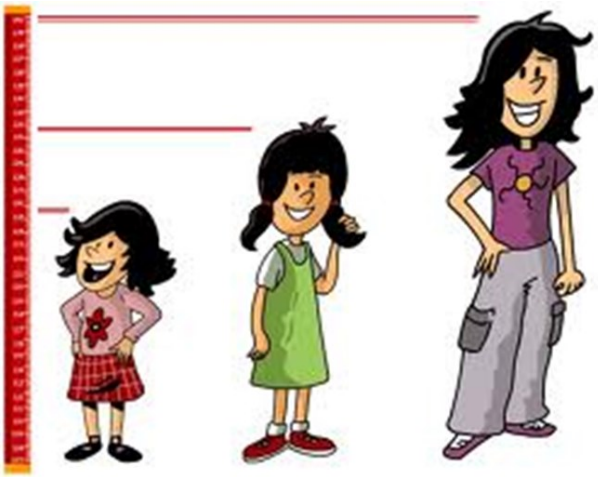


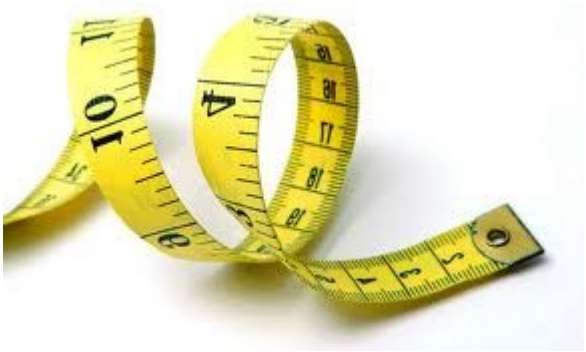



One centimeter

One millimeter

# The meter



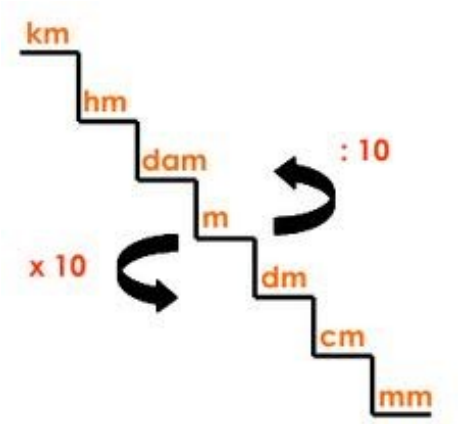


To measure the length and width of medium sized objects ( 1 – 2 meters length) we use a tool called sartorial meter or simply meter.

To measure longer distances we can use measuring roulette.

Some distances are to large to measure them in meters, so we use its multiples. They are:

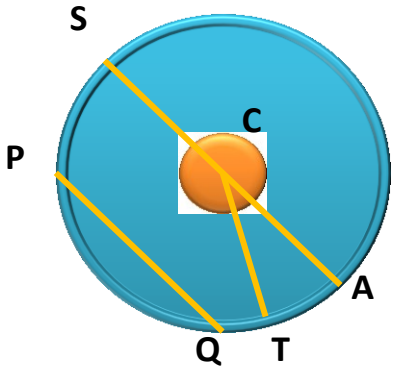
1 decameter is equal to 10 meters: 1 dam = 10 m.
1 hectometer is equal to 100 meters: 1 hm = 100 m.
1 kilometer is equal to 1000 meters: 1 km = 1000 m.
1 milliammeter is equal to 10,000 meters: 1 mam = 10000 m




Make conversions:

| | |
|---|---|
| **3 hm =** | |
| **8 dam =** | |
| **7 km =** | |
| **5 mam =** | |
| **2 dam =** | |
| **1 hm =** | |
| **4 km =** | |
| **6 mam =** | |

# Circumference and circle




- A disc has a circle shape.
- The edge of the disc has circumference shape.
- Point **C** is the center of the circle.
- **CT** segment is a **radius** of the disc.
- The radius joins the center with a point on the circumference.

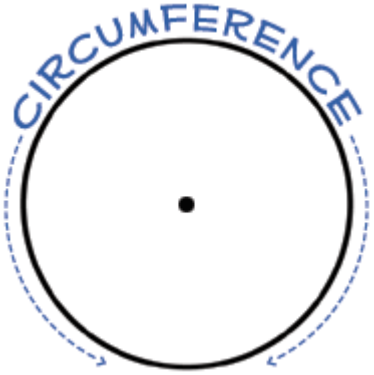
- The ST segment is the diameter of the circle. The diameter is the length of the line segment joining two points on the circle passing through the center.
- Note that the diameter is two radios.
- Point **C** is the center of the circle.
- The segment **PQ** is a chord of the circle. A rope connects two points on the circle.
- Note that the diameter is the largest chord of the circle.

The diameter of a circle is twice of the radius.

$$d = 2 \times r$$

## Circle Perimeter

Take a rope and put it around a circular plate.
Make a mark to recognize the amount of rope used.
Stretch the rope and measure the marked segment.
It is the perimeter of the circle.



# Grids

Grids are widely used in science, engineering, even for building puzzles. But we want to use the for copying purposes, when initial images can be copied one-to-one, can be enlarged or reduced.

## So we will:

Put initial figure in a rectangle, measuring its sizes and trying to use multiples of 1 centimeter. In general case when we want to use squares, take exact measures of sides in millimeters and choose one of the common multiples as a size of a square., thus dividing each side in several segments of equal size (look at the next page).

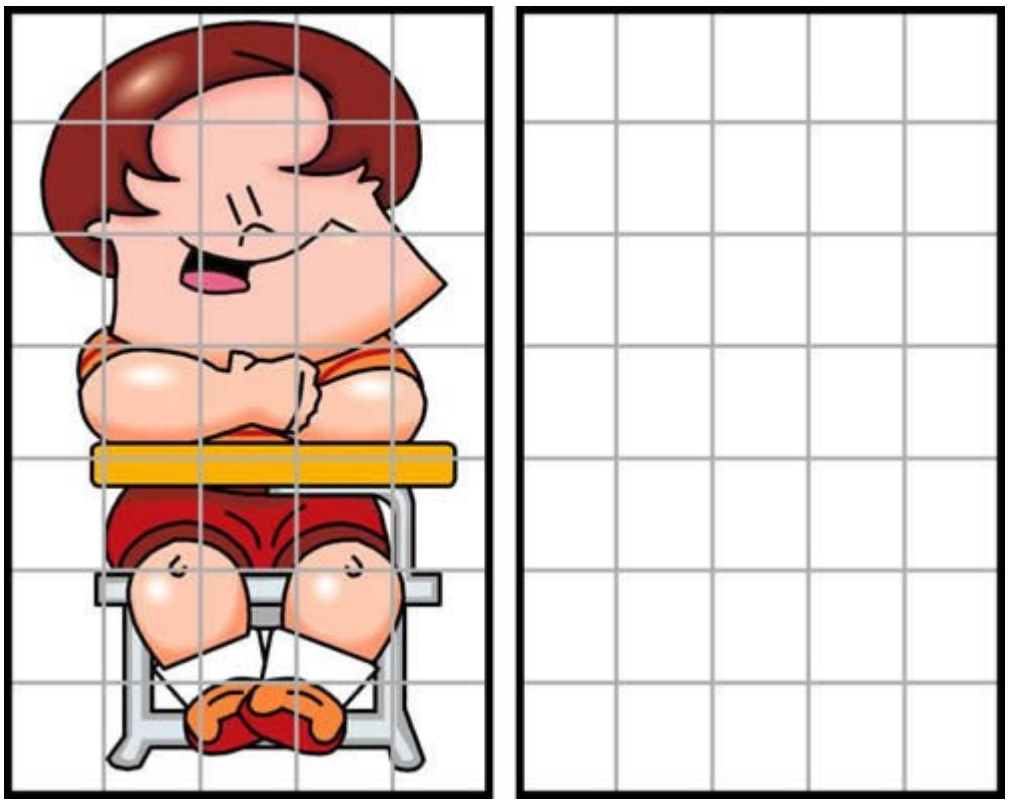
Neatly draw a grid over a picture or photo.

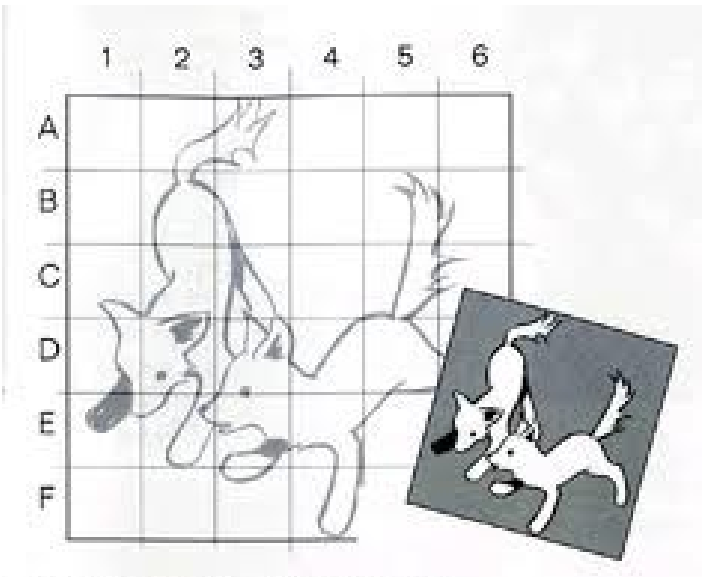
In the sheet where you want to put a copy, draw another grid with the same number of squares.
If we want to enlarge the drawing, the size of the square should be greater than in the initial grid.
If, however, we want to reduce the size of the figure, the size of new segment should be smaller.

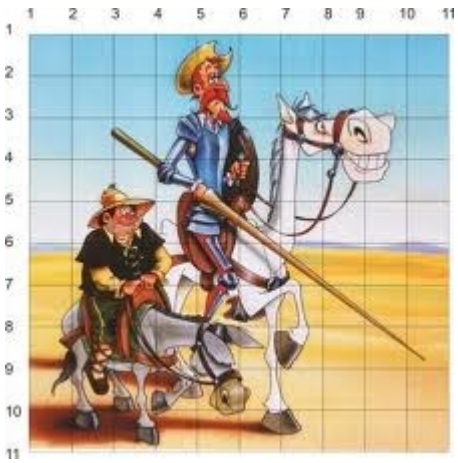
Look at the initial picture, look closely at each line or stroke of the figure and draw (retrace) them square by square (frame by frame) to get the complete picture.

The relation between lengths of the new and initial pictures gives a coefficient of multiplication (reduction).

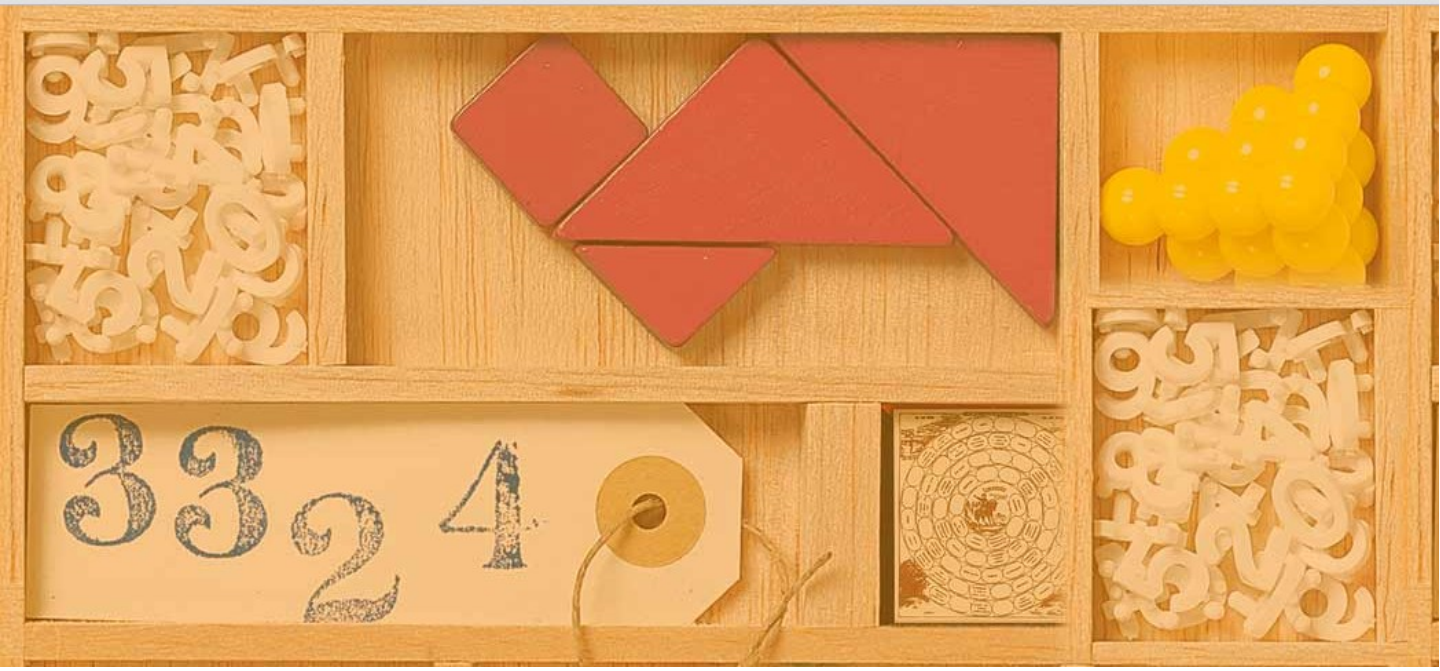
For squaring a rectangle is sufficient to find LCD of the side lengths and draw the square whose side are multiples of that number.




1 2 3 4 5 6
A
B
C
D
E
F


1 2 3 4 5 6 7 8 9 10 11
1
2
3
4
5
6
7
8
9
10
11

# Data records



***Data***: Are the numbers or measurements collected as a result of some observations.

It is necessary to organize (order) available information on certain criteria (characteristic), to take additional advantage by saving access time or storage space. Usually in ascending or descending order.

The frequency of event is the number of times event occurred in experiment or was found in recorded data.

The mean or arithmetic average is the result from adding all the values of the data set and dividing this sum by the total number of frequencies.

| Heights | Frequencies |
|---|---|
| 124 – 129 | 8 |
| 130 – 134 | 6 |
| 135 – 139 | 1 |
| 140 – 144 | 4 |
| 145 – 150 | 1 |
| | 20 |

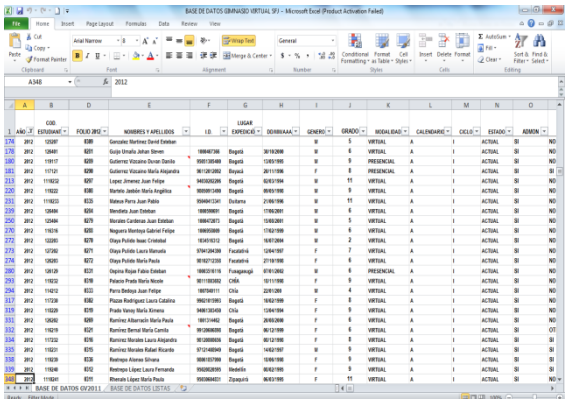
*For example:*

If the set is : **3, 5, 7, 3, 9, 3, 8, 4, 2**

The mean is : $\frac{3+5+7+3+9+3+8+4+2}{9} = \frac{42}{9} = 4.66\ldots$

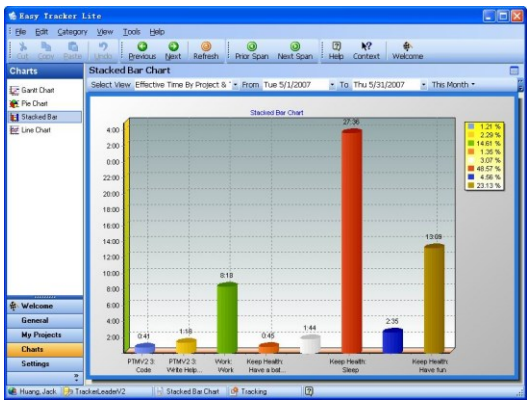
**Average:** $\frac{\text{Sum of values}}{\text{Quantity of values}}$

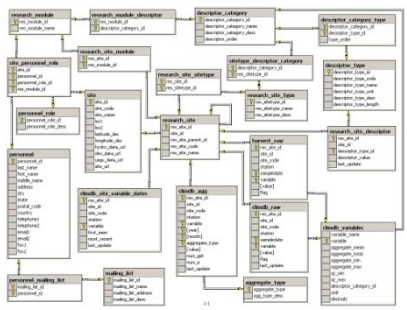
You can use technology to make an excellent record of data.



Programs like Excel, Access and tables in Word, Power Point and other programs, will be a great tool when making records. These programs are specialized and designed for this task.

Try it in Excel. Construct a small table of data records, find the average and use the chart tools to create bar chart.



*Data Base*